Warm-up!

A) At what point does the terminal ray of $\theta = \frac{4\pi}{3}$ intersect a circle of radius 10?

B) The terminal ray of the angle ϕ passes through a circle at the point $\left(2\sqrt{3}, -6\right)$. Find $\tan\phi$ and find the radius of the circle.

Advanced Trig Day 5:
(3.13) Polar Coordinates

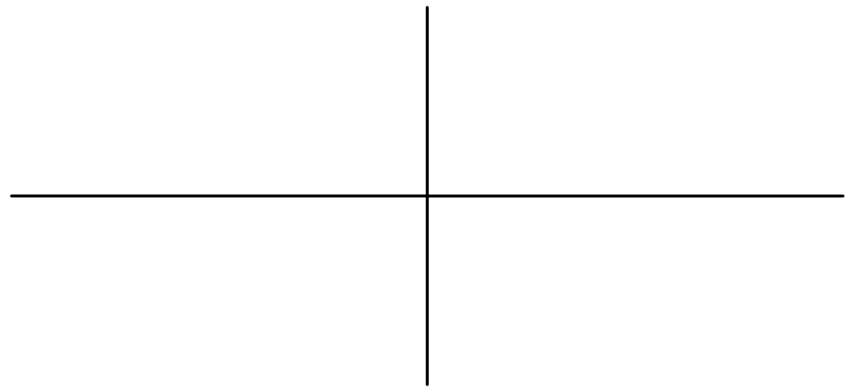
Homework: Polar Coordinates

Next Quiz

- Solving Trig Equations
- Solving Inverse Trig Equations
- Trig Identities
- Quiz is on Day 1 - Day 4

Polar Coordinates

Rectangular Coordinates: (x, y)



Polar Coordinates: (r, θ)

Polar Coordinate Example

A. $\left(3, \frac{\pi}{4}\right)$

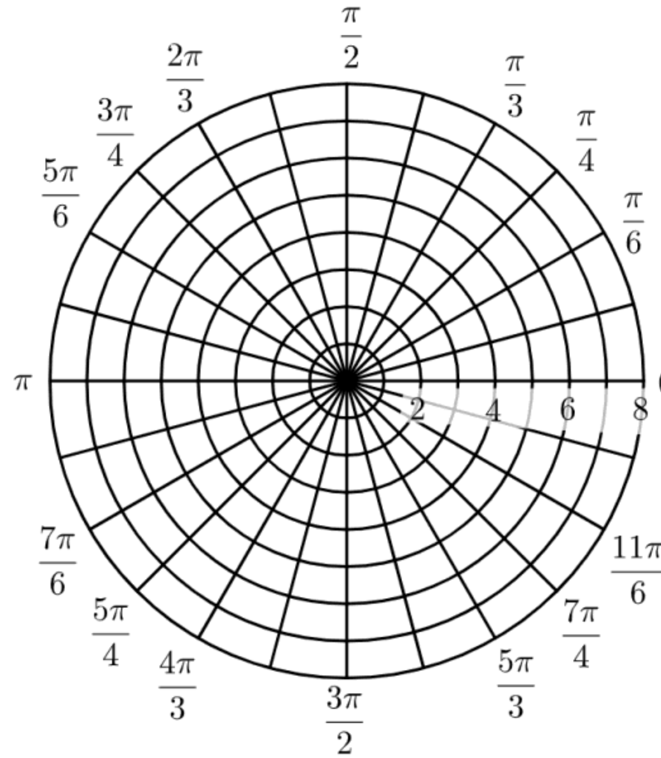
B. $\left(5, \frac{7\pi}{6}\right)$

C. $\left(4, -\frac{\pi}{3}\right)$

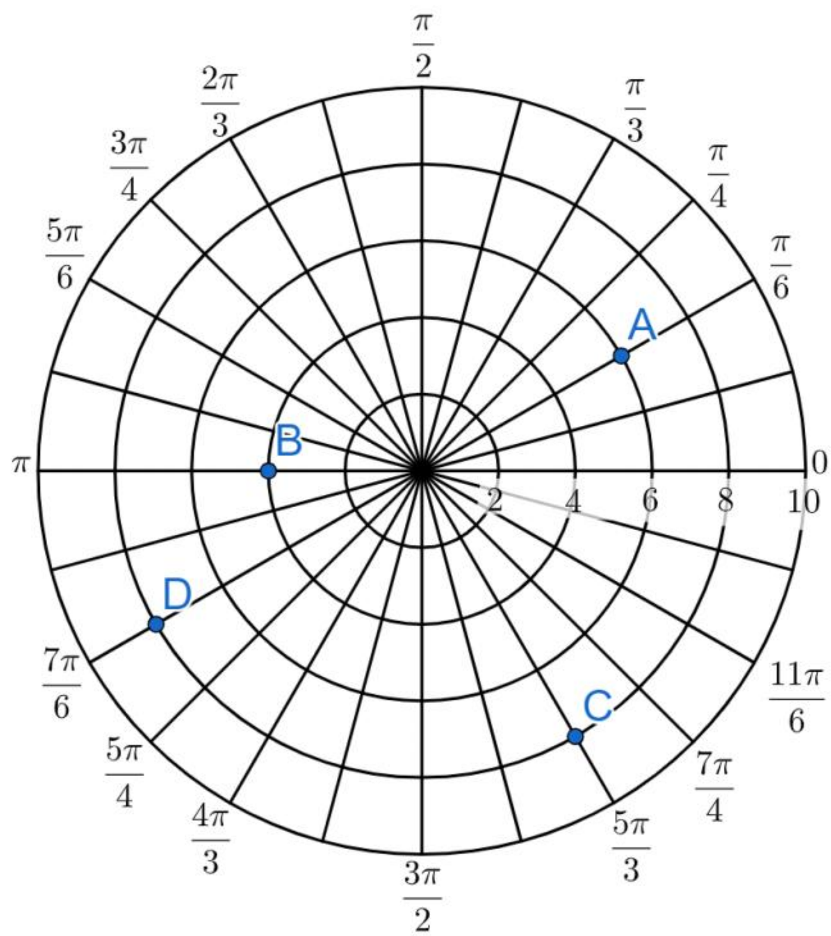
D. $\left(-6, \frac{\pi}{2}\right)$

E. $\left(4, -\frac{4\pi}{3}\right)$

F. $\left(3, -\frac{7\pi}{4}\right)$



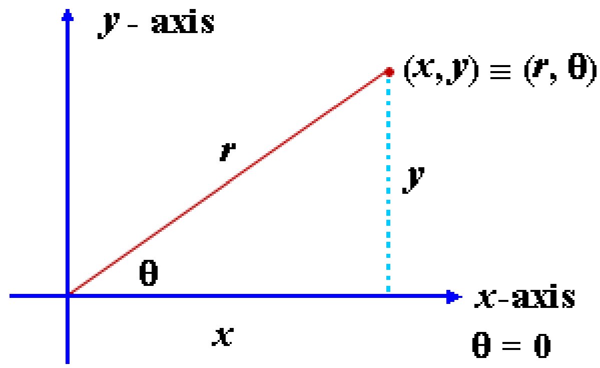
Find three ways of labelling each point with polar coordinates



Which is not the same as the others?

$\left(7, \frac{4\pi}{3}\right)$ $\left(-7, \frac{\pi}{3}\right)$ $\left(-7, -\frac{4\pi}{3}\right)$ $\left(7, -\frac{2\pi}{3}\right)$

Converting Polar Coordinates to Rectangular



Since $\cos(\theta) = \frac{x}{r}$ …

Since $\sin(\theta) = \frac{y}{r}$ …

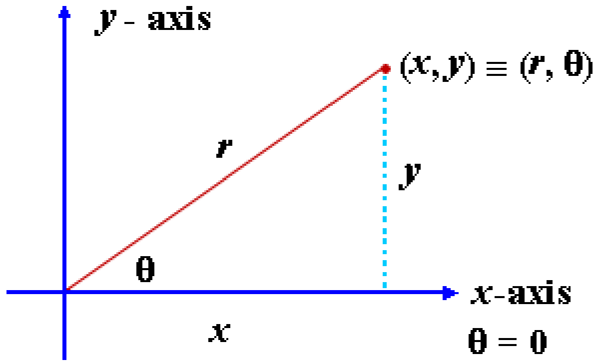
Convert from polar to rectangular

$$\left(5, -\frac{\pi}{3}\right)$$

$$\left(-4, \frac{3\pi}{4}\right)$$

$$\left(6, \frac{\pi}{2}\right)$$

Converting Rectangular Coordinates to Polar



$$r^2 = x^2 + y^2$$

$$\tan(\theta) = \frac{y}{x}$$

Converting from rectangular to polar

A. $(3, -3)$

B. $\left(-\sqrt{3}, 1\right)$

Converting from rectangular to polar

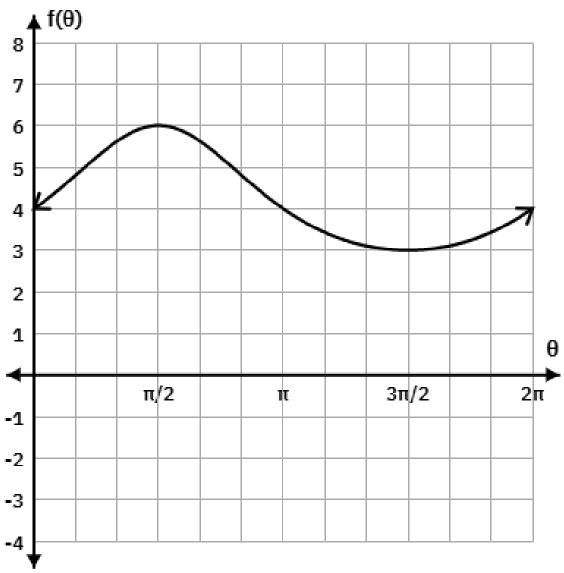
C. $(0,10)$

D. $(-3,-4)$

E. $(-3,4)$

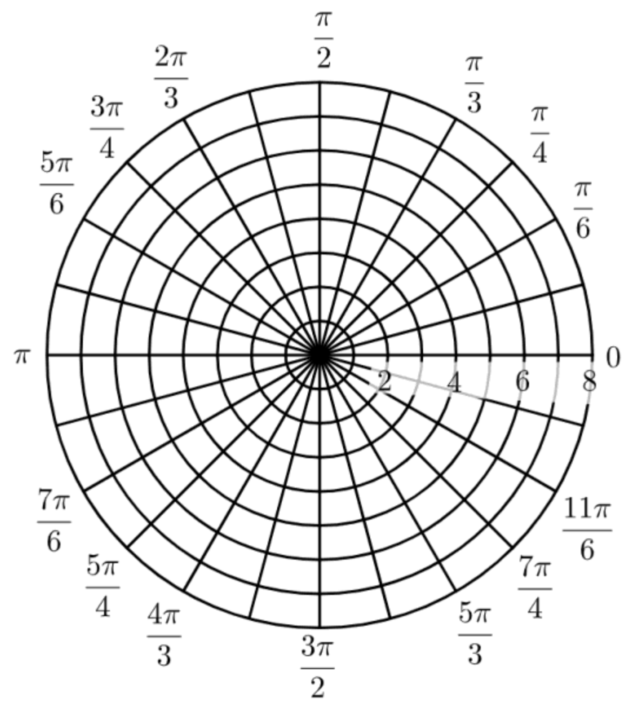
We are used to seeing the relationship between two variables illustrated with a rectangular graph. But we can also illustrate the relationship between two variables with a polar graph.

$$f(\theta) = \frac{12}{3 - \sin\theta}$$



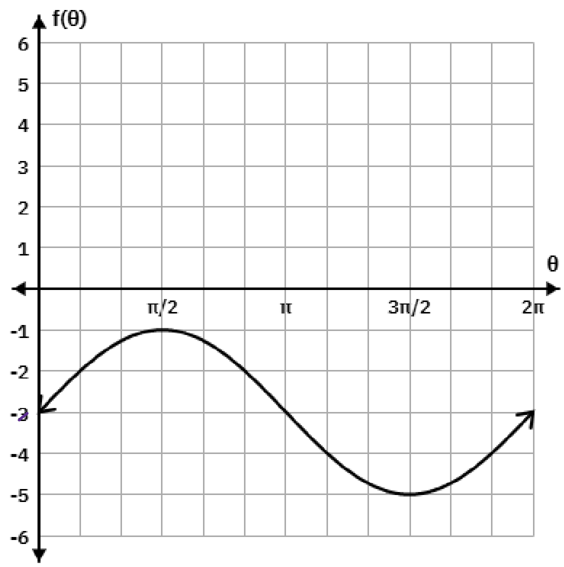
θ	$f(\theta)$

$$r = f(\theta)$$



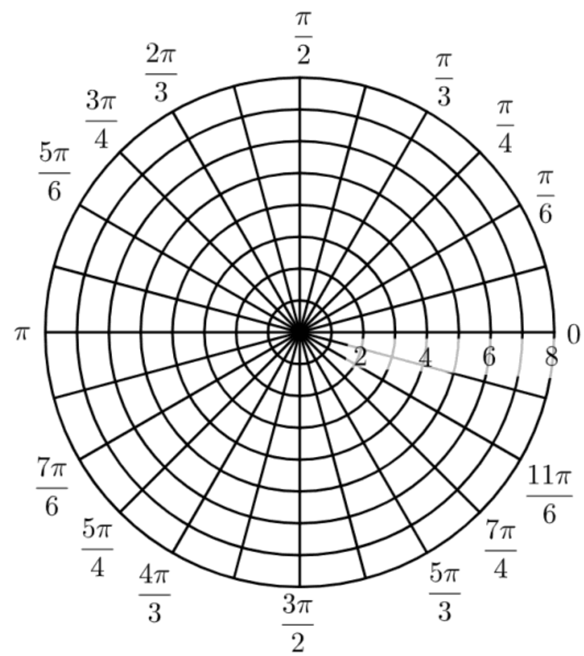
We are used to seeing the relationship between two variables illustrated with a rectangular graph. But we can also illustrate the relationship between two variables with a polar graph.

$$f(\theta) = -3 + 2\sin(\theta)$$



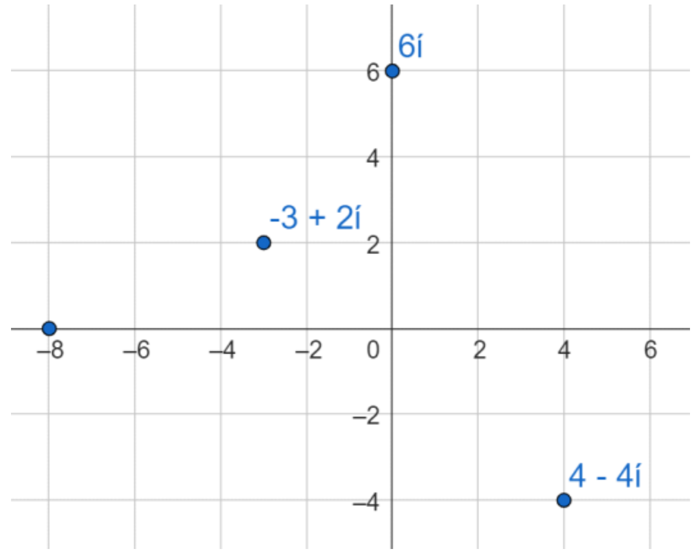
θ	$f(\theta)$

$$r = f(\theta)$$



Sometimes we like to treat complex numbers as points in the "complex plane"

The number $4 - 4i$ is sort of like



The point with rectangular coordinates $(4, -4)$

AKA the point with polar coordinates $\left(4\sqrt{2}, \frac{7\pi}{4}\right)$

Real part of the number is always the x-value of the point

Imaginary part of the number is always the y-value of the point

Let $z = 6 + 2\sqrt{3}i$

Find the rectangular coordinates of the point that corresponds with the value of z

Find the polar coordinates of the point that corresponds with the value of z

Let $z = 6 + 2\sqrt{3}i$

Rather than write z as $\left(6, 2\sqrt{3}\right)$ or as $\left(4\sqrt{3}, \frac{\pi}{6}\right)$, we sometimes write it as

Rectangular

$$\left(4\sqrt{3}\cos\left(\frac{\pi}{6}\right)\right) + i\left(4\sqrt{3}\sin\left(\frac{\pi}{6}\right)\right)$$

Polar

Let's call this the "polar form" of the complex number.

What's nice about this is that it's easy to quickly see what the rectangular coordinates or the polar coordinates of the point are.

If the point corresponding with a complex number has polar coordinates (r, θ), then we write the polar form of the complex number as

$$(r\cos\theta) + i(r\sin\theta)$$

Practice FRQ 4 Focused on Advanced Trig

Practice
FRQ 4 Foc...

FRQ 4 Practice Focused on Advanced Trig

Practice FRQ 4 Problem Alligator

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 2^{x-5}$$
$$h(x) = \arctan(x+1)$$

i. Solve $g(x) = 12$ for all values of x in the domain of g.

ii. Solve $h(x) = -\frac{\pi}{6}$ for all values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \ln(2x^3) - \ln(10x^4) - 6\ln x$$
$$k(x) = \frac{1 - \sin^2 x}{\sin\left(x + \frac{3\pi}{2}\right)}$$

i. Rewrite $j(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln$(expression).

ii. Rewrite $k(x)$ as a single term involving $\sec(x)$ and no other trigonometric function.

(C) The function m is given by $m(x) = \cos^2(3x) + 2\cos(3x)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem Baboon

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
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- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 2\log_4(x) + 7$$
$$h(x) = 6\sin^2 x$$

i. Solve $g(x) = 8$ for all values of x in the domain of g.
ii. Solve $h(x) = 3$ for all values of x in the interval $[0, 2\pi)$.

(B) The functions j and k are given by

$$j(x) = \frac{4 \cdot 8^x}{2^{4x-1}}$$
$$k(x) = (\sec^2 x)\left(\frac{1}{2}\sin(2x) + \cos^2 x\right)$$

i. Rewrite $j(x)$ in the form $a(b)^x$.
ii. Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

(C) The function m is given by $m(x) = \arccos(\tan x)$. Find all input values in the domain of m that yield an output value of $\frac{\pi}{2}$.

Practice FRQ 4 Problem Crocodile

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \log_4(3x)$$
$$h(x) = 4\sin^{-1}(x+1)$$

i. Solve $g(x) = -1$ for values of x in the domain of g.

ii. Solve $h(x) = \cos^{-1}\left(-\frac{1}{2}\right)$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = -\log_2(x-2) + 4\log_2(x+1) - \log_2(5)$$
$$k(x) = \frac{\cot^2 x}{\sin^2 x + \cot^2 x + \cos^2 x}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

ii. Rewrite $k(x)$ as a power of one trigonometric function.

(C) The function m is given by $m(x) = \sin(2x) - \sin(x)$. Find all input values in the domain of m that yield an output value of 0.

Practice FRQ 4 Problem Dolphin

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 6 \arcsin x$$
$$h(x) = 2\log_4 x - \log_4 3$$

i. Solve $g(x) = -2\pi$ for values of x in the domain of g.
ii. Solve $h(x) = \log_4(12)$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = 3\log_2(x+5) - 1$$
$$k(x) = \frac{\cot x}{1 + \cot^2 x}$$

i. Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.
ii. Rewrite $k(x)$ as a multiple of $\sin(2x)$. Your answer should be of the form $a\sin(2x)$.

(C) The function m is given by $m(\theta) = \cos(2\theta) + 9\sin\theta$. Find all input values in the domain of m that yield an output value of -4.

Practice FRQ 4 Problem Elephant

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \ln(3x - 5)$$
$$h(x) = 2\tan^{-1}\left(\frac{x}{2}\right)$$

i. Solve $g(x) = 2$ for values of x in the domain of g.

ii. Solve $h(x) = \arccos\left(\frac{1}{2}\right)$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \csc\left(x + \frac{\pi}{2}\right)$$
$$k(x) = \frac{\sin(x - \pi)}{\left(\cos\left(x - \frac{\pi}{2}\right)\right)^2}$$

i. Rewrite $j(x)$ as an expression involving $\cos x$ and no other trigonometric functions.

ii. Rewrite $k(x)$ as an expression involving $\csc x$ and no other trigonometric functions.

(C) The function m is given by $m(x) = \sqrt{3}\cot\left(x - \frac{\pi}{2}\right)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem Frog

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = 5(4)^x$$
$$h(x) = \cos\left(x + \frac{\pi}{2}\right)$$

i. Solve $g(x) = 2$ for values of x in the domain of g.

ii. Solve $h(x) = -\frac{\sqrt{3}}{2}$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.

(B) The functions j and k are given by

$$j(x) = (\sec^2 x)(\cos(2x))$$
$$k(x) = \frac{27 \cdot 9^x}{\sqrt{3^x}}$$

i. Rewrite $j(x)$ as an expression involving a power of $\tan x$ and no other trigonometric functions.

ii. Rewrite $k(x)$ as an expression of the form $3^{(ax+b)}$

(C) The function m is given by $m(x) = \sqrt{3}\cos(2x)$. Find all input values in the domain of m that yield an output value of $\frac{3}{2}$.

Practice FRQ 4 Problem Giraffe

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \tan(3x)$$
$$h(x) = 4e^{2x} + 5e$$

i. At what values of x does the graph of $y = g(x)$ have vertical asymptotes?

ii. Solve $h(x) = 6e$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \frac{1}{3}\log x - 2\log x$$
$$k(x) = 2\sin\left(x - \frac{\pi}{4}\right) - \sqrt{2}\sin x$$

i. Rewrite $j(x)$ as a single logarithm without negative exponents in any part of the expression. Your result should be of the form $\log(\text{expression})$.

ii. Rewrite $k(x)$ as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

(C) The function m is given by $m(x) = 4\csc(4x) - \sin(4x)$. Find all input values in the domain of m that yield an output value of 3.

Practice FRQ 4 Problem Hippopotamus

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

(A) The functions g and h are given by

$$g(x) = \tan^2 x$$
$$h(x) = \log_5(2x - 1) - \log_5(x - 2)$$

i. Solve $g(x) = 3$ for values of x in the interval $\left[\frac{\pi}{2}, \pi\right)$.

ii. Solve $h(x) = 2$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \frac{(2^x \cdot 8^{x+1})^2}{4^x}$$
$$k(x) = \frac{4\cos^2 x - 2\cos(2x)}{\cot^2 x - \csc^2 x}$$

i. Rewrite $j(x)$ as an expression of the form 2^{ax+b}.

ii. $k(x)$ is equivalent to a constant value. Determine the value of this constant.

(C) The function m is given by $m(x) = 4\tan^{-1}(\sin(2x))$. Find all input values in the domain of m that yield an output value of π.

Answers to FRQ 4 Practice Focused on Advanced Trig

On the AP Exam, work must be shown where applicable.

Practice FRQ 4 Problem Alligator

(A)

i. $x = 5 + \log_2 12$

ii. $x = -\frac{\sqrt{3}}{3} - 1$

(B)

i. $\ln\left(\frac{1}{5x^7}\right)$

ii. $-\frac{1}{\sec x}$

(C) $x = \frac{2\pi k}{3}$, where k is any integer

Practice FRQ 4 Problem Baboon

(A)

i. $x = 2$

ii. $x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \frac{5\pi}{4}, x = \frac{7\pi}{4}$

(B)

i. $8\left(\frac{1}{2}\right)^x$

ii. $\tan x + 1$

(C) $x = \pi k$, where k is any integer

Practice FRQ 4 Problem Crocodile

(A)

i. $x = \frac{1}{12}$

ii. $x = -\frac{1}{2}$

(B)

i. $\log_2\left(\frac{(x+1)^4}{5(x-2)}\right)$

ii. $\cos^2 x$

(C) $x = \pi k, x = \frac{\pi}{3} + 2\pi k, x = -\frac{\pi}{3} + 2\pi k$ where k is any integer

Practice FRQ 4 Problem Dolphin

(A)

i. $x = -\frac{\sqrt{3}}{2}$

ii. $x = 6$

(B)

i. $\log_2\left(\frac{(x+5)^3}{2}\right)$

ii. $\frac{1}{2}\sin(2x)$

(C) $\theta = \frac{7\pi}{6} + 2\pi k$, $\theta = \frac{11\pi}{6} + 2\pi k$ where $k \in \mathbb{Z}$

Practice FRQ 4 Problem Elephant

(A)

i. $x = \frac{e^2+5}{3}$

ii. $x = \frac{2\sqrt{3}}{3}$

(B)

i. $\frac{1}{\cos x}$

ii. $-\csc x$

(C) $x = \frac{2\pi}{3} + \pi k$, where k is any integer

Practice FRQ 4 Problem Frog

(A)

i. $x = \log_4\left(\frac{2}{5}\right)$

ii. $x = \frac{\pi}{3}$

(B)

i. $1 - \tan^2 x$

ii. $3^{\left(\frac{3}{2}x+3\right)}$

(C) $x = \frac{\pi}{12} + \pi k$, $x = \frac{11\pi}{12} + \pi k$, where k is any integer

Practice FRQ 4 Problem Giraffe

(A)

i. $x = \frac{\pi}{6} + \frac{\pi}{3}k$, for $k \in \mathbb{Z}$

ii. $x = \frac{1}{2}\ln\left(\frac{e}{4}\right)$

(B)

i. $\log\left(\frac{1}{x^{\frac{5}{3}}}\right)$

ii. $-\sqrt{2}\cos x$

(C) $x = \frac{\pi}{8} + \frac{\pi k}{2}$, for any integer k

Practice FRQ 4 Problem Hippopotamus

(A)

i. $x = \frac{2\pi}{3}$

ii. $x = \frac{49}{23}$

(B)

i. 2^{6x+6}

ii. -2

(C) $x = \frac{\pi}{4} + \pi k$, $k \in \mathbb{Z}$